Scheme of Work 2020-2021 Subject: Further Mathematics						
Year Group: Year 13 **Specification:** **A level**						
Lesson No	**Topic & Objectives**	**Key Activities & Specialist Terminology**	**Big Think Qs & Stretch**	**Assessment** **(Include relevant GCSE Q stem)**	**Home work**	**Lit** **Num** **SMSC Codes**
	PROOF -understand that various types of proof can be used to give confirmation that previously learnt formulae are true, and have a sound mathematical basis; -understand that there are different types of proof and disproof (e.g. deduction and contradiction), and know when it is appropriate to use which particular method;	• Manipulating the LHS of a result and using logical steps (normally algebraic) to make it match the RHS or vice versa (or, sometimes, manipulating both sides to reach the same expression). • Manipulating an expression to show it holds true for all values. For example, an inequality can always be ≥ 0 if we manipulate the LHS to be in the form of $[\text{something}]^2$ since anything squared will always be bigger or equal to zero. This argument can be used on a gradient function to prove a function is increasing.	Introduce using areas and the expansion of $(a + b)^2$ to prove Pythagoras' theorem as an example of using a logical sequence of steps in order to deduce a familiar result. Link with Trigonometry (Unit 6d) and provide a deduction of the compound-angle formula for e.g. $\sin (A + B)$		Exam style questions from the book and from maths	

		• using an appropriate proof within other areas of the specification later in the course			genie. com	
2	ALGEBRAIC AND PARTIAL FRACTIONS -be able to simplify algebraic fractions by fully factorising polynomials up to cubic - be able to split an proper and improper fraction into partial fractions, dividing the numerator by the denominator (by polynomial long division or by inspection).	1 Showing the reverse process where a simplified rational function is split into two (or more) partial factions 2. Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only) **1.1** 3. Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear)	Exam questions tend to focus on factorising polynomials and then cancelling common factors to simplify algebraic fractions. For example: Simplify $\frac{x^2-5x-6}{x^2-10x+24} \div \frac{x^2-x-2}{x^2-4x}$ Examples of each of the following types need to be covered. Linear: $\frac{5x-5}{(x+3)(x-2)}$ $\frac{2}{x^2-1}$ $\frac{7x+3}{x(x+1)}$ Repeated: $\frac{4x^2-3x+5}{(x-1)^2(x+2)} \equiv \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{(x+2)}$ Improper: $\frac{2x^2+5x-6}{(2x-1)(1+x)} \equiv A + \frac{B}{2x-1} + \frac{C}{1+x}$			

	FUNCTIONS AND MODELLING Objectives:	1.Sketching graphs of functions involving modulus functions; 2. Solving equations and inequalities involving modulus functions	Display the graph of $y = \|2x - 1\|$ and discuss this with students, drawing			

	- understand what is meant by a modulus of a linear function - be able to work out the domain and range of functions - use functions in modelling, including consideration of limitations and refinements of the models	3. Working out the composition of two functions 4. Understanding the condition for an inverse function to exist.	comparisons with the 'non-modulus' graph and making sure everyone recognises that $y = \|2x - 1\|$ does not have any negative values of y (the graph 'bounces up' with the x-axis acting like a mirror). Ask questions such as: When does the function machine fail to find an inverse? Do any functions have a self-inverse? Is an inverse function always possible?			
	SERIES AND SEQUENCES Objectives: - be able to use the standard formulae associated with	Knowing the difference between convergent and divergent sequences Knowing the proofs and derivations of the sum formulae (for both AP and GP) Knowing that a sequence can be generated using a formula for the *n*th	Start by recapping the work students did on sequences at GCSE (9-1) Mathematics before moving on to the new A level content, paving the way for the sigma notation in the following section.			

	arithmetic series and sequences; - be able to use the standard formulae associated with geometric series and sequences	term or a recurrence relation of the form $x_{n+1} = f(x_n)$; • Understanding how a recurrence relation of the form $U_n = f(U_{n-1})$ can generate a sequence;	Use practical situations, for example involving money, to illustrate APs and GPs and contrast the different ways they grow. Cover questions in which sequences can be used to model a variety of different situations. For example, finance, growth models, decay, periodic (tide height for example) etc.			
	THE BINOMIAL THEOREM - be able to find the binomial expansion of $(1 + x)^n$ for rational values of *n* and $\|x\| < 1$	Understanding and use the binomial expansion of $(a + bx)^n$ for rational *n*, including its use for approximation; be aware that the expansion is valid for $\left\|\frac{bx}{a}\right\| < 1$ (proof not required).	Begin by reviewing the expansion of $(a + b)^n$ when *n* is a positive integer. Ask students to expand $(1 + x)^4$ and then try $(1 + x)^{-2}$. Why does it fail to work? Which coefficient calculation breaks down?			

	- be able to find the binomial expansion of $(a+bx)^n$ for rational values of *n* and $\left\lvert\frac{bx}{a}\right\rvert < 1$	Knowing how to use the binomial theorem to find approximations (including roots). Using partial fractions to write a rational function as a series expansion	If we consider the 'complicated' fraction below, it needs to be simplifies into two simpler fractions each of which only involve a *single* algebraic bracket. $\frac{2x^2+5x-10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}$ We can now rewrite each term as a binomial series. (It is important to demonstrate that the $\frac{B}{x-1}$ term will become $B(x-1)^{-1}$.)			
	Trigonometry -be able to derive and use the formulae for arc length and area of sector. -be able to solve identities involving sec, cosec and cot	Understanding the definition of a radian and be able to convert between radians and degrees; Knowing and be able to use exact values of sin, cos and tan;	A good introduction is to ask the class to work out sin (30 + 60)°.It is equal to sin (90)° = 1. Go on to ask whether sin 30° + sin 60° gives the same value (either using a			

	- be able to use double angle identities to rearrange equations into a different form and then solve	Knowing and be able to use the identities $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \text{cosec}^2 x$ to prove other identities and solve equations in degrees and/or radians. Using compound angle identities to rearrange expressions or prove other identities; Solving equations of the form $a\cos\theta + b\sin\theta = c$ in a given interval.	calculator or using surds). They should discover that the values are different. Explain that the reason for this is that you can't simply multiply out functions in this way Derive and cover examples using half angle formulae by adapting the double angle versions.			
October Half Term						
	PARAMETRIC EQUATIONS Objectives:	Understanding and using the parametric equations of curves and conversion between Cartesian and parametric forms. Using parametric equations in modelling in a variety of contexts	Begin by explaining the difference between the Cartesian system, when a graph is described using $y = f(x)$, and the parametric system, which uses $x = f(t)$			

	- be able to convert between parametric and Cartesian forms - be able to plot and sketch curves given in parametric form	Recognising some standard curves in parametric form and how they can be used for modelling.	and y = g(t) for some parameter t. Illustrate this by asking the class to consider $x = 5t$ and $y = 3t^2$ and to try to eliminate t from the two equations. This will give $y = \frac{3}{25}x^2$ or $25y = 3x^2$. (This is a quadratic equation – parabola.) Repeat for $x = 5t$ and $y = \frac{5}{t}$. This becomes $y = \frac{25}{x}$ (a hyperbola).			
	DIFFERENTIATION Objectives -be able to find the derivative of $\sin x$ and $\cos x$ from first principles.	Using the chain rule and the formula $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, initially u can be given to students	Explain that if $y = 2e^x$ then $\frac{dy}{dx} = 2e^x$.			

	- be able to differentiate functions involving e^x, $\ln x$ and related functions such as $6e^{4x}$ and $5 \ln 3x$ and sketch the graphs of these functions - be able to differentiate using the product and the quotient rule	Finding and identify the nature of stationary points and understand rates of change of gradient. Knowing how to model the growth or decay of 2D and 3D objects using connected rates of change;	Encourage students to lay work out carefully, using correct notation and $\frac{dy}{du}$ and $\frac{du}{dx}$, not always $\frac{dy}{dx}$.			
	NUMERICAL METHODS Objectives - be able to locate roots of $f(x) = 0$ by considering changes of sign of $f(x)$;	Understanding the principle of iteration; Appreciainge the need for convergence in iteration;	The method at A level is to consider the roots of the function $y = f(x)$ as the intersection of the two functions $y = x$ and $y = f(x)$ (hence $x = f(x)$). Use an iteration of the form $x_{n+1} = f(x_n)$ to find a root of			

	-be able to use numerical methods to find solutions of equations.	Understanding how the Newton-Raphson method works in geometrical terms Using numerical methods to solve problems in context.	the equation $x = \mathrm{f}(x)$ and show how the convergence can be understood in geometrical terms by drawing cobweb and staircase diagrams			
	Integration 1 -be able to integrate expressions by inspection using the reverse of the chain rule (or function of a function); - be able to use trigonometric identities to manipulate and simplify expressions to a form which can be integrated directly.	Integrate x^n, (including $\frac{1}{x}$) and integrate e^{kx}, sin kx , cos kx and related sums, differences and constant multiples Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$. Integration to include integrating functions defined parametrically Students should recognise integrals of the form $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\mathrm{d}x = \ln \lvert \mathrm{f}(x) \rvert + c$.	Students must have lots of practice at working with logarithms and exponentials when integrating, and should leave their answers in exact form. They also need to be fluent in knowing the key trig identities and how to manipulate them from the ones in the formula book.			
Christmas Holidays						