Scheme of Work 2020-2021						
Subject: Further Mathematics **Specification: Edexcel** **Level: AS / Yr 1 A Level**						
Lesson No	**Topic & Objectives**	**Key Activities & Specialist Terminology**	**Big Think Qs & Stretch**	**Assessment (Include relevant GCSE Q stem)**	**Home work**	**Lit Num SMSC Codes**
	COMPLEX NUMBERS (21h) Objectives: -be able to add, subtract and multiply complex numbers in the form *x* + i*y* with *x* and *y* real understand and be able to use the laws of indices for all rational exponents; -be able to use and interpret Argand diagrams -be able to convert between the Cartesian form and the modulus-argument form of a complex number be able to construct and interpret simple loci in the Argand	1. Define i as $\sqrt{-1}$. Hence $i^2 = -1$ 2. Use the difference of two squares and surds (rationalisation) to illustrate the method of manipulation of complex conjugates. 3. Highlight the fundamental theorem of Algebra and the nature of polynomials and roots**.** 4. Link Argand diagrams with **i**, **j** vectors. 5. Students should be encouraged to take care when drawing Argand diagrams, and to use an accurate scale	Take care when finding the complex conjugate of a number given in the form '(imaginary part)i + real part', e.g. 8i - 3. The complex conjugate is – 8i – 3, and not 8i + 3For surds the denominator is rationalised. For complex numbers the denominator is made real. Emphasise that zz* is real. Revisit the factor theorem To work out which region is required for the inequalities,	Class tests obtained from Edexcel website	Exam style questions from the book and from mathsgenie.com	

	diagram such as $\|z-a\| > r$ and $\arg(z-a)=\theta$		choose a point (for circles, the centre of the circle is usually a good choice) to find whether it is in the region required				
2	**SERIES (5h)** Objectives -be able to use sigma notation; understand and use formulae for the sums of integers, squares and cubes; -be able to use known formulae to sum more complex series	1 If arithmetic and geometric series in A level Mathematics has been covered, this could be reviewed, showing that $\sum r$ is an arithmetic series with $a=1$ and $d=1$. Knowledge that $\sum_1^n 1 = n$ is expected. 2.Practicing algebraic skills 3. Students must become fluent, and continue to develop thinking skills such as choosing an appropriate method, and interpreting the language in a question. 3. Generating results for a sum it is good practice to test it for small values of n. This does not prove it is correct, but if one value of n does not work, then it is obvious that the result is incorrect.	Using partial fractions and the method of differences to sum series involving fractions. $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$ The formulae for $\sum_{r=1}^{n} 1 = n$ and $\sum_{r=1}^{n} r = \frac{n}{2}(n+1)$ should be learnt. To find $\sum_{r=1}^{2n} r^2$, use the standard result $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$ and replace n by $2n$ to give $\sum_{r=1}^{2n} r^2 = \frac{2n}{6}(2n+1)(4n+1)$.				

	ALGEBRA AND FUNCTIONS (8h) Objectives: -be able to understand and use the relationship between roots	Using Vieta's formulas, which don't necessarily tell us what the roots are, but enable us to evaluate other expressions	Knowledge of the following identities will be helpful: $\alpha^2+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta$			

	and coefficients of polynomial equations up to quartic equations - be able to form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).	Starting with quadratic equations, for example, if α and β are roots of the equation $3x^2+5x-1=0$ find a quadratic equation that has roots of $\alpha+3, \beta+3$. Using sums and products of roots, this problem can be solved as follows: Using $\alpha+\beta=\frac{-5}{3}$ and $\alpha\beta=\frac{-1}{3}$, then the sum of new roots is $\alpha+3+\beta+3=\frac{-5}{3}+6=\frac{13}{3}$ and the product of new roots is $(\alpha+3)(\beta+3)=\alpha\beta+3(\alpha+\beta)+9=\frac{-1}{3}-5+9=\frac{11}{3}$. So the quadratic equation with roots $\alpha+3$, $\beta+3$ becomes $z^2-\frac{13}{3}z+\frac{11}{3}=0$, ie. $3z^2-13z+11=0$	• $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$ • $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ • $\alpha^3+\beta^3+\gamma^3=(\alpha+\beta+\gamma)^3-3\alpha\beta(\alpha+\beta)-3\alpha\gamma(\alpha+\gamma)-3\beta\gamma(\beta+\gamma)-6\alpha\beta\gamma$ Extend the solving method for quadratics to cubics and then quartics.			
	CALCULUS **(6h)** Objectives: -be able to derive formulae for and calculate volumes of revolution about both the *x* and *y*-	1. Both volume for rotation about the *x*-axis $V=\pi\int y^2\,\mathrm{d}x$ and the *y*-axis $V=\pi\int x^2\,\mathrm{d}y$ are required. Stress the importance of quoting the general formula for volume before substituting values.	Finding the volume of a cone. Finding the volume of a sphere.			

	axes.	Look at using more complex functions for *y* or *x* so when squaring will challenge and develop their integration skills	Finding the volume of a section rotated about the axes. Subtracting one volume from the other. Torus? Finding the volume of shapes rotated around a line that is parallel to one of the axes			
October Half Term						

1-2	**MATRICES** **(26h)** Objectives: -be able to add and subtract matrices of the same dimension; -be able to multiply a matrix by a scalar; be able to multiply conformable matrice - be able to calculate the inverse of non-singular 2×2 and 3×3 matrices. -be able to use matrices and their inverses to solve linear simultaneous equations, including three linear simultaneous equations in three variables -be able to use inverse matrices to reverse the effect of a linear transformation; -be able to use the determinant of a matrix to determine the area scale factor of a transformation; -be able to find invariant points and lines for a linear transformation.	Students should show that they understand the process of inverting a matrix and the change in complexity of inverting a 2×2 to a 3×3. Adding dimensions to a matrix add significant complexity to find the inverse. Generally should be using a calculator to find the inverse of a 3×3 Students will need to know the exact trigonometric ratios for sin, cos, tan of 0°, 30°, 45°, 60°, 90°. To calculate the coordinates of a point after a transformation, multiply the transformation matrix by the coordinates. To identify a 2D transformation from its matrix, consider how the points (1, 0) and (0, 1) in the form of the matrix $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ are transformed	Investigating $(A^{-1})^{-1} = A$, $(kA)^{-1} = 1/k\ A^{-1}$, $(A^n)^{-1} = (A^{-1})^n$ where n is a positive integer. Investigate the determinant properties. Matrix Algebra. How to solve matrix equations. AB = C using the inverse. Permutation matrices. $\mathbf{A}^n$ matrices. Prove that for all 2×2 matrices **A**, **B** and C (**AB**)**C** = **A**(**BC**). Show that if **A** (1×2), **B** (2×3) and **C** (3×4) then (**AB**)**C** = **A**(**BC**)			

	PROOF **(6h)** Objectives -be able to obtain a proof for the summation of a series, using induction; -be able to use proof by induction to prove that an expression is divisible by a certain integer; -be able to use mathematical induction to prove general statements involving matrix multiplication.	Students need to demonstrate their understanding of the concept of proof by induction, and not just learn the appropriate statements. Give examples of the three different types of proof covered in this specification, with particular care given to proving that an expression is divisible by a certain integer, as this is the type students will typically find the most challenging. Use mathematical induction to produce a proof for a general term of a recurrence relation. Proof by Strong Induction.	1 Prove the statement is true for $n = 1$, ie, show that when $n = 1$, LHS = RHS. 2 Assume the statement to prove IS true for $n = k$ (where k is a positive integer). This is just a matter of rewriting the original statement with n replaced by k. 3 Write an expression for the next term i.e. $n = k + 1$. 4 Manipulate this expression and simplify it to look like the original expression with a $(k + 1)$ replacing all the k's. 5 This has proved that the situation works for k +1 if it works for k. 6 Therefore, it can be induced that it would work for $k + 2$, $k + 3$, etc. in a similar way. Hence it works for all positive integers k.			

5-6	**VECTORS** **(21h)** Objectives -know how to find the vector equation of a line in both two and three dimensions; -understand and use the Cartesian forms of an equation of a straight line in three dimensions; -understand and use the vector and Cartesian forms of the equation of a plane. -be able to use the scalar product to express the equation of a plane; -be able to use the scalar product to calculate the angle between two lines; -be able to use the scalar product to calculate the angle between two planes; -be able to use the scalar product to calculate the angle between a line and a plane.	Students should be familiar with both the vector form (**r** = **a** + λ**b**) and the Cartesian form $\left(\frac{x-a_1}{b_1}=\frac{x-a_2}{b_2}=\frac{x-a_3}{b_3}\right)$ of the equation of a straight line. Show similarities with $y = mx + c$ and **r** = **a** + λ**b**. The scalar form of the equation of a plane (**r•c** = p) should be familiar. The formulae for angle between two lines: $\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{\|a\|\|\mathbf{b}\|}$, the angle between two planes: $\cos\theta = \left\|\frac{\mathbf{n}_1\cdot\mathbf{n}_2}{\|n_1\|\|\mathbf{n}_2\|}\right\|$ and the angle between a line and a plane: $\sin\theta = \left\|\frac{\mathbf{b}\cdot\mathbf{n}}{\|\mathbf{b}\|\|\mathbf{n}\|}\right\|$, are all similar, but they need to be learnt. The intersection of line and planes The intersection of planes and the system of 3 solutions and the use of matrices and technology to help solve these. The emphasis of unique solution, an infinite solution and no solutions. The perpendicular distance between two lines or between a point and a line or a plane gives the shortest distance between them. The perpendicular distance from (α, β, γ) to $n_1x + n_2y + n_3z + d = 0$ is $\frac{\|n_1\alpha+n_2\beta+n_3\gamma+d\|}{\sqrt{n_1{}^2+n_2{}^2+n_3{}^2}}$. This is given in the formula booklet	Proof of the Cosine rule using vectors and that it is uses the scalar product. Perpendicular distance between 2 planes with 2 common points. Extend the perpendicular distance to look at distance between 2 planes. Could bring in here the cross product.			